LOCAL AND GLOBAL LINEAR KARREN INSTABILITY

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Abstract thin films, morphological instability.

Linear karren are dissolution patterns, carved into inclined limestone slabs (fig. 1a). They appear as arrays of regularly spaced rills, oriented along the slope and present wavelengths, ranging from millimetres to several metres [1]. While their hydrodynamical origin is undisputed, the mechanics at play behind their formation remain elusive. The pattern's wavelength selection and streamwise extent are two questions of particular interest.

One point of view on their origin involves the destabilisation of a run-off film flow over a soluble substrate. Indeed, it was experimentally observed that uniform film flows over inclined gypsum substrates carve streamwise streaks [2]. An instability mechanism in the coupling between the flow velocity and the dissolution rate was proposed: over a range of transverse wavenumbers, the flow is accelerated in the perturbation's troughs compared to the crests, causing a thinning of the solutal boundary layer and enhancing the dissolution. Soon after, a local linear instability analysis on the laminar film flow over a soluble substrate was performed, confirming the mechanism [3].

On one hand, the obtained local dispersion relations present a plateau of almost equally unstable modes (fig. 1b), which hinders the predictive power of the analysis. On the other hand, interpreting the resulting locally varying temporal growth rates is challenging. We thus perform a global, two-dimensional, linear instability analysis. We exploit the separation between the geological time scale of the dissolution process and the hydrodynamical time scale of the flow, in order to formulate a quasi-stationary eigenvalue problem, in which the flow and concentration fields adapt instantaneously; the eigenmodes are then contained only in the substrate topography perturbation. While the local instability mechanism remains pertinent, the resulting global dispersion relation has a far more peaked dominant wavenumber region, holding promise for future validation.

(a) (b)

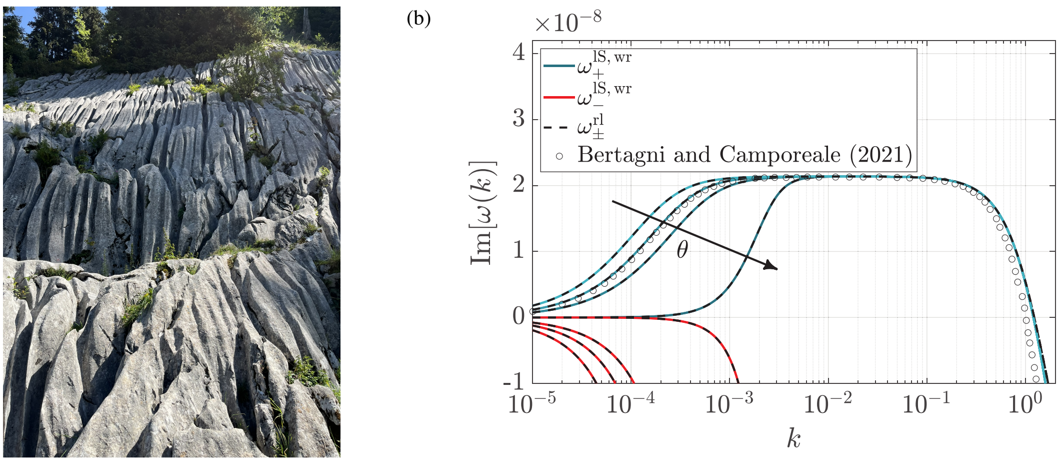

Figure 1: (a) Linear karren near Stockhorn in the Bernese Alps. (b) Sample local dispersion relations for varying inclination angle with respect to the horizontal $\theta = \{\pi/8, \pi/4, 3\pi/8, \pi/2\}$: dimensionless growth rate $\mathrm{Im}[\omega]$ as a function of the dimensionless wavenumber k. The circles represent Bertagni & Camporeale's (2021) result [3], while the lines follow several explicit dispersion relations obtained through boundary-layer models [4].

References

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