Kapitza instability of Nusselt's condensation film

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A quiescent, saturated vapour condenses onto a uniformly cooled, inclined plate and forms a laminar, incompressible, gravity-driven film flow of Newtonian liquid of constant material properties (fig. 1). We set out to investigate the linear stability of Kapitza waves on this film's free surface [1]. We perform our analysis in the framework of the one-sided model, whereby we consider the vapour phase as mechanically passive [2].
In Nusselt's pioneering theory, the effect of condensation is twofold: with respect to energy conservation, through its associated latent heat, and, crucially, with respect to mass conservation, where it acts as a source. This makes the basic flow one of a spatially developing nature – the film's thickness scales as the streamwise position to the power of *1/4* [3].
We make the problem dimensionless by rescaling all distances with a characteristic length H_N, which we choose as the equality of the streamwise position with the film thickness at that position (fig. 1); we use a hydrostatic pressure scale, a viscous velocity scale from Poiseuille's law, and recast the temperature field from *0* on the wall to *1* on the interface.
As regards the local linear stability, *i.e.* in the quasi-parallel approximation, condensation appears to damp perturbations. Since thinner liquid layers promote heat transfer, vapour tends to condense more in the perturbation's throughs rather than on the crests, thus stabilising the system. The threshold for the onset of local linear instability corresponds to larger Reynolds numbers, comparing the flow's inertia and viscosity, than in the classical thin-film flow, without condensation.
A higher Jakob number, measuring sensible to latent heat, which characterises a cooler plate and, hence, improved condensation, increases the critical Reynolds number. Nevertheless, it also ensures that the basic flow thickens – and reaches this local instability criterion – faster, ultimately shortening the critical streamwise distance for the emergence of waves.
Thanks to the convective nature of the instability, the spatial problem is well-posed. We seek to establish the system's response to a harmonic forcing of real angular frequency ω. The linearised, quasi-parallel perturbation evolution equations are a quadratic eigenvalue problem for the wave number k, the solution

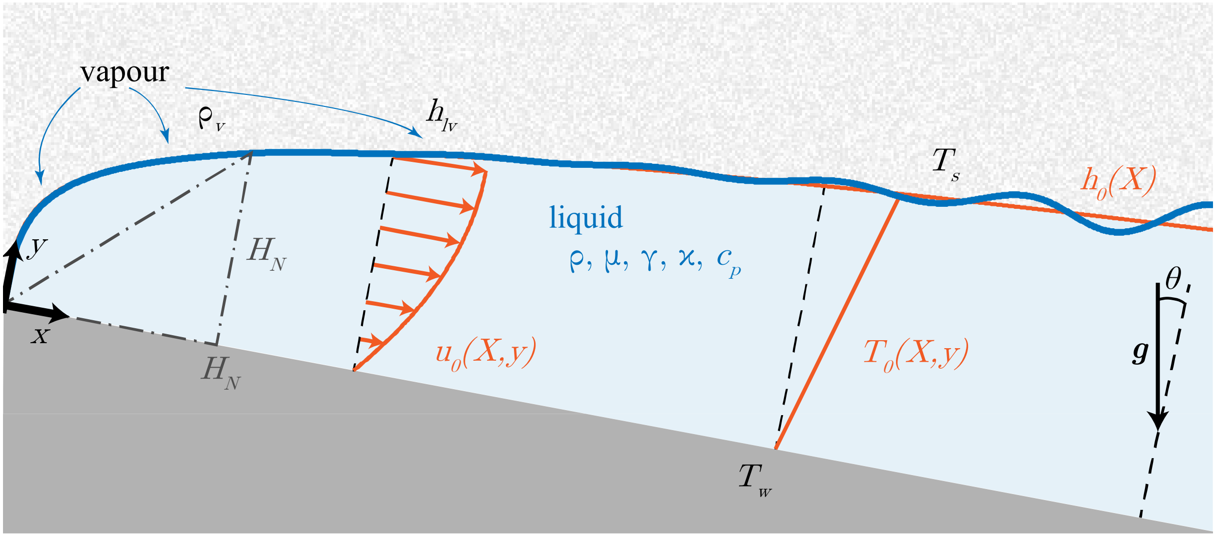

FIG. 1 A sketch of the problem at hand. The basic flow is represented in orange.

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[2] A. Oron, S. H. Davis, S. G. Bankoff, *Long-scale evolution of thin liquid films*, Rev. Mod. Phys. **69**(3), 931 – 980 (1997).
[3] W. Nusselt, *The surface condensation of water vapour*, Z. Ver. Dtsch. Ing. **60**(27), 541 – 546 (1916).

to which produces the spatial dispersion relation. For waves propagating in the direction of the flow, the negative imaginary part of the wave number is the spatial growth rate, and vice versa. It turns out that only one spatial branch, associated to downstream-propagating waves, becomes locally linearly unstable. This is the spatial Kapitza instability.

The analysis also shows that the forcing frequency, to which the system is locally most receptive, increases downstream, while the associated wave number remains almost constant. Furthermore, a separation of scales between these perturbation wavelengths and the basic flow's evolution length makes the weakly non-parallel approach suitable. We introduce a "slow" streamwise coordinate X, which captures the basic flow's streamwise dependence, and integrate the locally computed, quasi-parallel spatial dispersion relations along it (fig. 2). Before reaching the critical streamwise distance, the perturbations decrease in magnitude (see inset). This critical distance is longer for larger forcing frequencies. After the onset of instability, the perturbations grow in a sub-exponential manner, as the local spatial growth rates decrease downstream. One by one, larger forcing frequencies take precedence. Thereafter, following the WKB method [4], a first order expansion in the slow streamwise coordinate produces a correction to these spatial gains, which we compare to a global resolvent analysis, similar to Viola *et al.* (2016) for the growth of instabilities in spatially developing swirling wakes [5]. This enables us to predict the linearly most amplified inlet forcing frequency.

The approach can also be extended to other weakly non-parallel thin film flows, for instance a rain-fed falling film, the thickness of which scales as the streamwise position to the power of *1/3*.

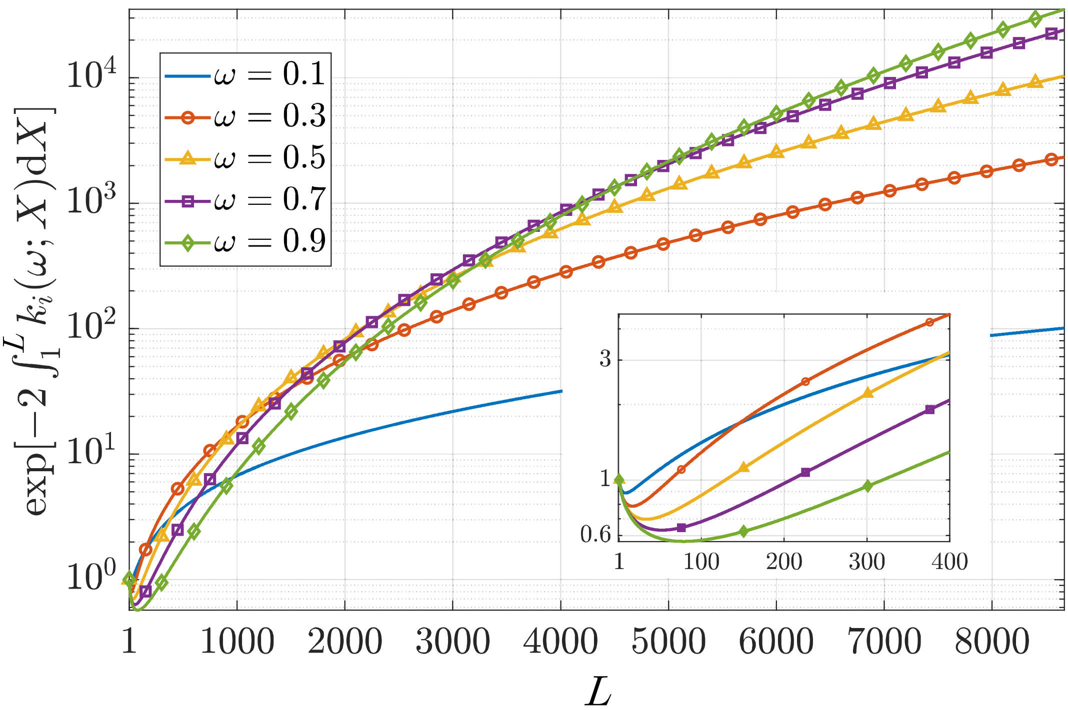

FIG. 2 Spatial gain. Material properties of water, inclination angle $\theta = \pi/4$, Jakob number $Ja \approx 18$, describing a cooling of *10*K. $\omega = 0.1$ corresponds to around $80\mathrm{s}^{-1}$ and the largest dimensionless plate length $L \approx 8700$ scales to *1*m.

[4] P. Huerre, M. Rossi, *Hydrodynamic instabilities in open flows*: "Hydrodynamics and Nonlinear Instabilities", edited by C. Godrèche and P. Manneville, 81 – 294 (1998).

[5] F. Viola, C. Arratia, F. Gallaire, *Mode selection in trailing vortices: harmonic response of the non-parallel Batchelor vortex*, J. Fluid Mech. **14**(790), 523 – 552 (2016).